Using PRODAS 6 DoF Traj Simulation to Estimate Dispersion

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We frequently receive requests to estimate dispersion of a projectile at specific ranges based on a defined error budget. A typical error budget for two shots is shown in Figure 1.

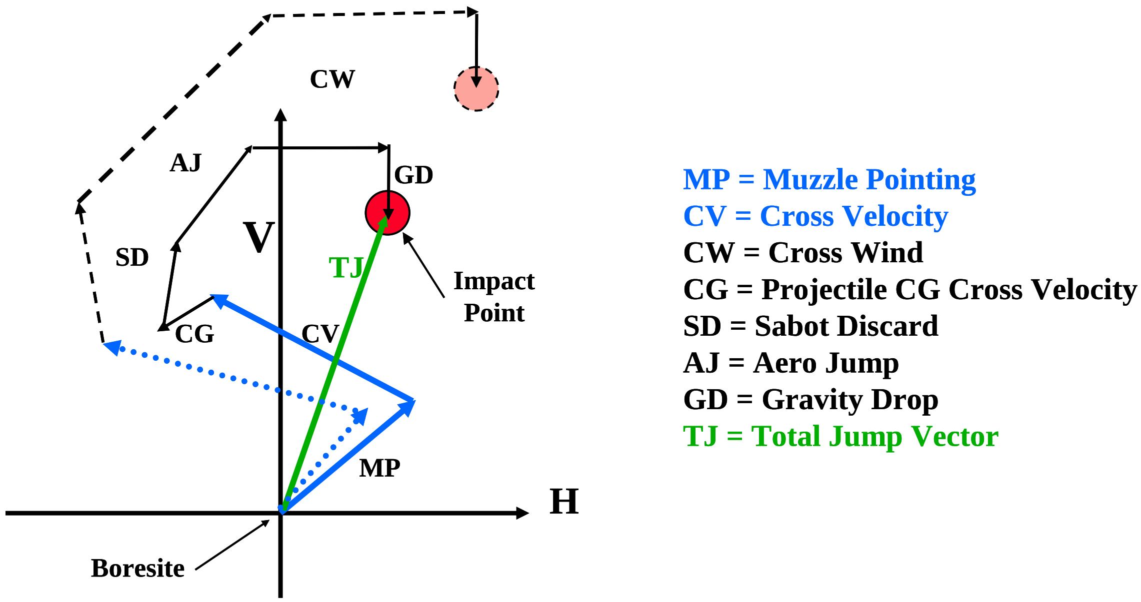

Figure 1: Error Budget Example

For direct fire applications versus a vertically oriented target, it is quite simple to use the fixed plane 6 DoF trajectory simulation in PRODAS to generate a series of sensitivity factors as a function of range, multiply by each of the appropriate sized error factors, and add the components up in a root-sum-square fashion to estimate the total dispersion of the system at any range desired.

In Figure 1, the barrel related components are shown in blue, while the dispersion components related to the projectile are in black. The barrel related components, muzzle pointing and cross velocity, are typically fairly small, provided the in-bore transit time of the projectile is reasonably repeatable (standard deviation of approximately 0.3 msec or less). For this reason, and because the barrel related dispersion components are typically much smaller than the other components once the bullet has travelled a few hundred yards (meters) down range, we typically ignore these components of the dispersion error budget.

The effect of the remainder of the error budget elements on dispersion can be simulated relatively simply, by running a series of 6 DoF trajectory "perturbations" and subtracting those results from a "baseline" 6 DoF trajectory. The trajectory perturbations we typically run are:

- + 50 rad/sec Pitch Rate
- + 50 rad/sec Yaw Rate
- + 10m/sec muzzle velocity
- + 2% drag
- + 10 m/sec cross wind
- + 10 m/sec head wind

When performing the perturbation trajectory simulations, the user must be mindful to "restore" the perturbed initial conditions to their baseline values when running subsequent simulations. The tabulated output information is copied to a spreadsheet for subsequent operations and summations.

Once the baseline and perturbation trajectory simulations have been completed, we generate what we call "sensitivity factors" in the horizontal and vertical plane at each output range by subtracting the Y and Z values for the baseline trajectory from the perturbed trajectory and dividing by the input perturbation value (e.g. divide by 50 for a 50 rad/sec input yaw rate) to get a sensitivity factor in inches or mm per unit perturbation factor. The generated sensitivity factors are then multiplied by the estimated error magnitude for each dispersion component (1 sigma) to generate an error at range for each dispersion source. An example of the dispersion estimation procedure is shown below.

I typically put the one standard deviation error budget values at the top of a spread sheet work sheet.

	1 σ value	
Angular rate	10	rad/sec
Cross Wind	2.37	m/s
Head Wind	2.37	m/s
Drag Variability	1.5	Percent
MV Variabiilty	20	FPS
Range Estimation	25	m
Aim Error	0.05	mils

Table 1: Typical Error Budget Component Values

I then start with a "baseline" 6 DoF trajectory simulation which has output increments I desire and also has an initial gun elevation which allows the projectile to cross the maximum range of interest a few meters above the ground (e.g. Z at last range between 5 & 10 meters). I then run a series of 6 DoF simulations with the "bulletized" perturbations shown above, and subtract these results from the "baseline" simulation to generate "sensitivity factors in the Y & Z axes as a function of range. Table 2 shows the Horizontal Cross Wind Sensitivity factor and the Vertical Jump sensitivity factors in the two right hand columns.

1.5 m/sec X Wind (90 deg wind direction)								
Time	X	Y	Z	Slant	Velocity		Cross Wind Sens Factor: cm/1 m/sec Cross Wind	Vertical Jump Factor, cm/ 1 m/sec Cross Wind
sec	m	cm	cm	m	m/sec		Y drift Sens. Factor	Z Alt. Sens Factor
0.0000	0.00	0.00	0.00	0	865		0.00	0.00
0.1190	99.93	0.41	366.54	100	816.18		0.36	0.23
0.2452	199.87	1.59	718.37	200	769.09		1.43	0.46
0.3793	299.82	3.62	1053.64	300	723.77		3.29	0.69
0.5218	399.77	6.55	1370.22	400	680.26		6.00	0.92
0.6735	499.72	10.43	1665.62	500	638.59		9.62	1.15
0.8353	599.69	15.33	1936.99	600	598.79		14.24	1.37
1.0079	699.66	21.31	2181.03	700	560.85		19.95	1.60
1.1922	799.64	28.41	2393.91	800	524.77		26.83	1.83
1.3893	899.63	36.70	2571.19	900	490.46		35.00	2.06
1.6005	999.63	46.24	2707.69	1000	457.63		44.55	2.29

Table 2: Cross Wind Sensitivity Factors

I then return the perturbated input to its original value and proceed to generate the next trajectory and compute horizontal and vertical sensitivity factors. This procedure is repeated until I have run trajectories for all the components in the dispersion error budget of interest.

Once this is complete, I collect the results of the product of the error budget factors multiplied by the sensitivity factors all into one area, and root-sum-square the error budget factor results as shown in Table 3.

Slant Range, m	Ang. Rate, H @ 10 rad/sec	Ang. Rate, V @ 10 rad/sec	X wind, H @ 2.37 m/s	X Wind, V @ 2.37 m/s	Head Wind, H @ 2.37 m/s	Head Wind, V @ 2.37 m/s	Drag, V @ 1.5 Percent	Muz. Vel., V @ 20 FPS	Range Error, V @ 25 m	Aim Error, H @ 0.05 mils	Aim Error, V @ 0.05 mils	Miss S.D., m
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
100	0.020	0.020	0.000	0.005	0.000	-0.005	0.000	0.000	0.022	0.005	0.005	0.037
200	0.040	0.040	0.005	0.016	0.000	-0.016	0.000	0.006	0.042	0.010	0.010	0.076
300	0.060	0.060	0.005	0.016	0.000	-0.016	0.000	0.012	0.063	0.015	0.015	0.111
400	0.080	0.080	0.005	0.021	0.000	-0.021	0.000	0.018	0.086	0.020	0.020	0.149
500	0.100	0.100	0.005	0.032	0.000	-0.032	0.000	0.030	0.108	0.025	0.025	0.189
600	0.120	0.120	0.005	0.037	0.000	-0.037	-0.008	0.043	0.126	0.030	0.030	0.226
700	0.140	0.140	0.005	0.042	-0.005	-0.042	-0.007	0.061	0.151	0.035	0.035	0.268
800	0.160	0.160	0.005	0.047	-0.005	-0.053	-0.007	0.079	0.172	0.040	0.040	0.309
900	0.180	0.180	0.005	0.053	-0.005	-0.058	-0.015	0.098	0.189	0.045	0.045	0.347
1000	0.200	0.200	0.011	0.063	-0.005	-0.063	-0.015	0.128	0.215	0.050	0.050	0.395

Table 3: Error Budget Components and RSS Miss

From here, you can plot up total miss vs. range or look at which factors comprise the largest portion of the total error budget (range error and angular rate induced dispersion in the example above).

Depending on your particular needs, you may want to compute the effects of bias errors and random errors separately.